

ASTRONOMY 9: HISTORY OF COSMOLOGY

Handout #19

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Special Relativity (1905)

- Albert Einstein (1879–1955)
 - Like Aristotle and Newton, created new conceptual foundation for physics
 - Worked on wide variety of problems (mechanics, gravitation, quantum and statistical physics)
 - Born Ulm, Germany; emigrated to U.S. in 1933; died Princeton, NJ
 - Detached childhood, strong distrust of all authority, but not an untalented student
 - “Person of the Century” (*Time*), socially engaged
 - Used his great celebrity to speak out on pacifism, democracy, Zionism; offered presidency of Israel
 - Science as path to self-transcendence; deep belief in Nature as a mystical riddle
- Motivation for Special Relativity
 - At age 16, Einstein thinks about “riding on a light beam”
 - If you catch up, light should appear to stop moving
 - But Maxwell’s theory says light *always* moves: no stationary EM waves allowed!
 - Ideas of length contraction and time dilation previously suggested by Lorentz and Fitzgerald to explain Michelson-Morley result, but did not understand why
- Postulates of Special Relativity
 - I. **Laws of physics are the *same* for all observers moving with constant velocities**
 - This is the old idea of (Galilean/Newtonian) relativity
 - Motion at constant velocity is *relative*
 - No experiment inside a train with closed windows can determine if it is moving at constant velocity
 - II. **The speed of light in empty space is always measured to be the *same*, regardless of motion of the source or observer!**
 - Very counter-intuitive: simple addition of velocities doesn’t work!
 - Maxwell is right, Newton wrong!
 - c is *absolute* (not everything is relative!)
 - From these simple postulates, all bizarre implications of relativity follow (thought experiments)
 - Effects are only noticeable when speeds approach $c = 300,000$ km/s, *far* removed from our everyday world
- Fun Implications of Special Relativity
 - I. **Simultaneity is *relative*!**
 - If two observers are moving relative to each other, two events that look simultaneous to one will *not* look simultaneous to the other!
 - Example: passenger stands in middle of moving train car and shines light forwards and backwards
 - * Both passenger and station observer measure c
 - * To passenger, hits front and back simultaneously
 - * To station observer, hits back of car first!
 - II. Time is relative: **time dilation!**
 - Moving observers will *not* agree that clocks tick at the same rate
 - Demolishes Newtonian idea of absolute time
 - Example: “light clock” in a moving train
 - * Tick/tock is light bouncing vertically off mirrors separated by height H

- * For observer moving with clock, tick/tock time is H/c
- * For station observer, the light has to go farther, so time is $> H/c!$
- * **Moving clocks run slower!** Factor is $\gamma = 1/\sqrt{1 - (v/c)^2}$ (“boost factor”)
- * Note reciprocity: observer on train will see that *station observer’s* clock runs slow
- * The fastest-running clock (“**proper time**”) is always the one that is moving with the observer
- * Note: you always perceive your *own* time “flowing” at the same “rate”

III. Space is relative: **length contraction!**

- Moving rulers become shorter along the direction of their motion!
- As with time dilation, factor is $\gamma = 1/\sqrt{1 - (v/c)^2}$
- Demolishes Newtonian idea of absolute space
- Example: decaying subatomic particles at speed close to c
 - * Typical half-life time τ for particle at rest
 - * Particles can live much *longer* when moving due to time dilation (observed all the time using *cosmic rays*)
 - * This means they can travel much *farther* than you would expect before decaying
 - * But imagine you are Superman, flying along with the particle
 - * Since particle is at rest in your frame, it decays in its usual short time
 - * But it *did* manage to cover the longer distance—how?
 - * Must be that space *contracted* along the direction of motion!

IV. Velocities do not simply add

- Throw a ball forward with speed u from a train moving with speed v
- Newton and Galileo: observer on ground sees speed $u + v$
- Einstein: observer on ground sees speed

$$\frac{u + v}{1 + uv/c^2}$$

- Note this is always $< c$, even if u and v are very close to $c!$
- Doppler shift is also more complicated:

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \sqrt{\frac{1 + v/c}{1 - v/c}}$$

V. Speed of light is the ultimate speed limit!

- In a second 1905 paper, $E = mc^2$
- Mass is a form of energy! Can convert from one to the other, for example:
 - * Nuclear fission (first uranium atomic bombs)
 - * Nuclear fusion (hydrogen bomb, powers the Sun)
 - * Even chemical reactions that give off energy result in a (tiny) change in mass
- Even when matter is at rest, still has some energy
- Speed something up, it has more energy, and therefore more mass: $m \propto \gamma$
- $v = c$ would lead to infinite mass, requiring infinite energy, so cannot be reached
- Any particle traveling at c must have zero “rest mass”
- Note: this is not simply a problem of engineering (like sound speed), but a fundamental physical principle!

• Twin “Paradox”

- Send one twin off to α Centauri and back
- Earth-bound will see the twin’s clock run slower, so when she gets back she will have aged less!
- But the traveler will see Earth’s clock run slower, so she will expect the Earth-bound traveler will have aged less
- Who is right?
- In fact the traveling twin ages less. Why?

- Only Earth-bound twin was in an inertial frame; in order to come back, traveler had to *decelerate*, turn around, and *accelerate*
- But Earth-bound *will* always see traveler’s clock run slow, so it is true that traveler ages less upon return
- Have to be careful to account for the acceleration when talking about what traveler will see
- Absolute Newtonian space and time are replaced by four-dimensional **spacetime**
 - Space and time not just a background arena, but active physical entities
 - Plot ct (time) vs. x (one dimension of 3-d space)
 - Particles at rest in this frame follow vertical lines
 - Particles in inertial (uniform velocity) frames follow *straight lines*
 - Light follows 45° diagonals
 - **Spacetime interval:** $\Delta s^2 = c^2\Delta t^2 - \Delta x^2$
 - * Note – sign makes this very different from Euclidean geometry!
 - * This interval is *invariant*: all observers will measure the same value for Δs (though different Δt and Δx)
 - Events outside the 45° **light cone** are “elsewhere” and cannot be in causal contact
- Remaining problem: how does gravity fit in with all this?
 - Need for a General Theory!