

ASTRONOMY 9: HISTORY OF COSMOLOGY

Handout #20

J. E. Baker
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General Relativity (1915)

- A Brief History of Gravity
 - Aristotle: natural tendency of motion towards/away from center of Earth
 - Kepler: a kind of magnetism
 - Descartes: cosmic vortices
 - Newton: mysterious instantaneous action at a distance, $F = GMm/d^2$
- Einstein fits **gravity** into the spacetime framework of relativity
- **General** theory of relativity
- The Principle of Equivalence (1907)
 - Imagine two situations:
 1. You are standing in a stationary elevator on Earth. All falling objects accelerate downward at $1g \approx 10 \text{ m/s}^2$.
 2. The same elevator is floating through space with no gravity, but the elevator is being *accelerated* upward at $1g$.
 - Einstein’s equivalence principle: there is *no* experiment you can do inside the elevator to distinguish these two situations! (Note: we are ignoring *tidal* forces, more on this later.)
 - Special relativity works in *freely falling* reference frames—inertial frames are *accelerating*
 - By transforming to freely falling frame, can get rid of gravity!
 - This expresses the equivalence of **inertial mass** m_i and **gravitational mass** m_g :
 - * Newton’s law of gravity: $F = GMm_g/d^2 = m_g g$
 - * Newton’s second law: $F = m_i a$
 - * Contrast with electric force $F \propto q_1 q_2 / d^2$: m_g is a gravitational “charge” analogous to q
 - * Electric charge has nothing to do with inertial mass; why then is $m_i = m_g$? No experiment has ever measured $m_i \neq m_g$.
 - * In Newtonian physics, this was an unexplained mystery.
 - * In general relativity, it implies that gravitation is simply *bent* spacetime!
- Curvature of Spacetime
 - Imagine two elevators floating through empty space
 1. Moving with constant velocity
 - * Inertial frame, thrown ball follows straight-line path
 2. Accelerating upward
 - * Non-inertial frame
 - * Ball thrown horizontally at speed v follows curved (parabolic) trajectory
 - * Imagine $v = c$: light beam also follows curved path!
 - By principle of equivalence, (2) is exactly the same as gravity \Rightarrow gravity bends light!
 - Or: gravity curves space and light follows the shortest path (“geodesic”) in the curved space
 - Gravity also curves time!
 - * Imagine clocks at center and edge of a disk rotating at high speed
 - * By special relativity, clock at edge ticks slower
 - * But this is an *accelerated* frame, again equivalent to gravity
 - * So **clocks in a gravitational field run slower**
 - Trampoline analogy for curved space
 - * Imagine a 2-d plane universe (would have to think in 4-d to visualize curved 3-d space!)
 - * Soccer ball follows straight-line motion on flat trampoline
 - * If you’re standing on the trampoline, the surface is *curved*

- * Soccer ball naturally follows curved path, could orbit you (if no friction)
- * Einstein's interpretation: Earth orbits Sun not because of *force* of gravity, but because it is following shortest path in curved space!
- Tidal forces
 - * A *uniform* gravitational field would be just like *tilting* the trampoline with no one standing on it: space is still *flat*, but everything accelerates in one direction
 - * Gravity is directed towards the *center* of massive objects, giving rise to tidal forces
 - * Objects on sides of elevator accelerate towards each other a little, since they're really accelerating towards Earth's center (not exactly "down")
 - * Usually small, but can be large (eg, falling into black hole)
 - * Curvature of space (departure from special relativity) really represents the tidal forces
- The Equations of G.R.
 - * Equation of the universe: $G^{\mu\nu} = 8\pi T^{\mu\nu}$
 - * Einstein: "I never realized that so many Americans were interested in tensor analysis."
 - * $G^{\mu\nu}$: **geometry**, curvature of 4-d spacetime
 - * $T^{\mu\nu}$: description of matter/energy densities, pressures, stresses, ...
 - * Matter tells space how to curve, and space tells matter how to move
- Mach's Principle
 - * Are you rotating, or is the universe rotating around you?
 - * Who decides what the inertial frames are?
 - * Ernst Mach's answer: the overall distribution of matter in the universe
 - * Mach's principle is mostly, but not fully, embodied within G.R.:
 - Spacetime curvature can exist independent of any mass (eg, gravitational waves)
 - Solutions to Einstein's equations for the geometry are not always unique for a given mass distribution
- Tests of the theory
 - Not easy, since differences between Newton and Einstein only become large for *very* strong gravity (like near a black hole)
 - 1. Arthur Eddington (1919) famously observed bending of starlight around the Sun during a solar eclipse
 - Deflection of only 1.75 arcseconds!
 - Einstein becomes popular hero
 - 2. Shift in perihelion (closest point to Sun) of Mercury's orbit
 - Newtonian gravity predicts 5514 arcseconds per century
 - Observed 5557 arcsec/century, 43 too much!
 - Agrees well with Einstein's GR prediction
 - 3. Gravitational waves
 - Propagate at speed c , no more Newtonian action at a distance!
 - Like ripples on a pond, fluctuations in curvature of spacetime
 - Still building large experiments which may detect them directly (LIGO)
 - Joseph Taylor (1993 Nobel Prize work): already detected *indirectly* because they carry energy away from the orbits of binary *pulsars* (mass of Sun but radius of S.F.!)
 - 4. Gravitational redshift
 - Clocks in stronger fields tick more slowly
 - So distant observer measures lower frequency for radiation coming out from inside a gravitational field, so longer wavelength (redshift)
 - Easily observed using spectra of *white dwarf* stars (mass of Sun but radius of Earth)
- Black Holes
 - Gravitational fields get stronger as objects become more massive and smaller
 - We know gravity deflects light; can imagine making an object so dense that not even light can escape! This is a black hole

- Schwarzschild showed that this happens if an object is smaller than $R_S = 2GM/c^2$
- For the Earth this is only 1 cm; for the Sun, about 3 km!
- Nothing can travel faster than light, so nothing inside the **event horizon** (E.H.) can have any causal influence on the outside
- For non-rotating black hole, event horizon is a sphere at radius R_S
- Light cones tilt sideways as you approach the black hole: space and time are interchanged!
- Observer on outside sees you fall slower and slower as you go in, infinite time to cross E.H.
- Black holes have almost certainly been discovered
 - * Large ones (10^6 - $10^9 \times$ mass of Sun) sit at the centers of many galaxies
 - * Smaller ones (1 - $10 \times$ mass of Sun) result from deaths of massive stars in supernova explosions
 - * Can't be seen directly, but matter falling in gets heated to very high temperature and emits high-energy radiation before crossing E.H.
- Wormholes might form, could they be used to zip across the universe?
 - * Probably not, they seem to be unstable and trajectories through them are spacelike (requires faster than light speed)
 - * Might be able to hold one open and go through, but only if *negative mass/energy* exists (pretty weird!)
- Global geometry
 - G.R. can describe not just local spacetime curvature (eg, due to Sun or galaxy), but *global* curvature of the whole universe!
 - Fifth postulate of Euclidean (ancient Greek) geometry:
 - * Given a line and a point not on the line, one and only one line can be drawn through the point so that it is parallel to the line
 - Some theorems: angles of triangle add to 180° , circumference of circle is $2\pi r$
 - If we modify the 5th postulate (but keep the other 4), we get interesting new *curved* geometries
 - Studied in late 1800s by Riemann and others
 - Three basic possibilities for a *homogeneous* universe:
 1. Accept Euclid's 5th postulate
 - * Space is "flat", ordinary geometry works
 2. Replace "one and only one line" with "no lines"!
 - * Example: surface of the Earth (again, have to think of 2-d universes in order to visualize curved space)
 - * "Straight lines" (shortest distance between two points) become *great circles* (lines of longitude, but **not** latitude)
 - * Different great circles always intersect twice, no parallel lines!
 - * Sum of triangle angles is $> 180^\circ$, $C < 2\pi r$
 - * Universe has **positive curvature**
 3. Replace "one and only one line" with "infinitely many lines"!
 - * Example: saddle (not very good analogy, only works near saddle point)
 - * Straight lines diverge, so can have infinite number of parallels
 - * Sum of angles is $< 180^\circ$, $C > 2\pi r$
 - * Universe has **negative curvature**
 - Note: we visualize these geometries by embedding 2-d space in a 3-d space, but in general the embedding is not necessary