

ASTRONOMY 9: HISTORY OF COSMOLOGY  
**Assignment #16—Solutions**

2000 May 1

1. Consider the following models for an expanding universe with differing mass–energy density  $\rho$  and cosmological constant  $\Lambda$ . For each model, describe the evolution of the universe in the past and future. Which do you prefer?

- (a)  $\rho > \rho_c$  and  $\Lambda = 0$

The high-density  $\Lambda = 0$  universe starts from a “bang” and collapses to a Big Squeeze in the future. It might be a one-time occurrence, or the universe might (somewhat improbably) go through repeated cycles of expansion and contraction.

- (b)  $\rho < \rho_c$  and  $\Lambda = 0$

The low-density  $\Lambda = 0$  universe starts from a “bang” and keeps expanding forever, with  $R \propto t$  in the future when the density becomes very low.

- (c)  $\rho = \rho_c$  and  $\Lambda > 0$

A critical-density  $\Lambda > 0$  universe starts from a “bang” and expands as  $R \propto t^{2/3}$  for a while. Eventually the repulsive  $\Lambda$  takes over and the expansion accelerates forever, with  $R \propto e^{Ht}$ .

- (d)  $\rho = \rho_c$  and  $\Lambda < 0$

If  $\Lambda$  is negative, it acts like an attractive force rather than a repulsive one. This universe starts from a “bang” and eventually turns around and collapses, because it eventually gets big enough that the  $\Lambda$  takes over and pulls everything back together (recall that  $\Lambda$  acts like a force which gets *stronger* with distance).

- (e)  $\rho = 0$  and  $\Lambda > 0$

This is the exponentially expanding de Sitter model, with  $R \propto e^{Ht}$  always. There is no beginning (the universe is eternal), and no end.

The class narrowly preferred (38%) the closed,  $\Lambda = 0$  model (a), which collapses at some point in the future. Better to end in fire than in ice, especially with the speculative possibility of a cyclical universe that is reborn like a phoenix from its ashes.

Some 31% of you voted for the  $\rho = \rho_c$  universe (c) with a positive cosmological constant (which evolves similarly to the universe we seem to actually live in, on current evidence). But note that this sort of universe may be a very bleak place to live in a few tens of billions of years—there is an *event horizon* (a region out of which we can never escape) that is *shrinking* with time, so that eventually we won’t even be able to see (much less get to) any other galaxies!

Another 23% voted for an open model with  $\Lambda = 0$  (b), which also expands forever, so we are not necessarily doomed in a Big Squeeze, nor trapped within a shrinking event horizon by  $\Lambda$ . But eventually the stars will burn out and the protons will probably decay, so it might still be kind of desolate.

The remaining 8% voted for the Einstein–de Sitter model ( $\rho = \rho_c$  and  $\Lambda = 0$ ), although this wasn’t actually one of the choices. Also expands forever, but more slowly than (b) or (c). The appeal here is that it’s the simplest theoretical model, with an amount of matter that is “just right”, and no funny  $\Lambda$ . This made it the favorite among cosmologists until observational evidence forced them to abandon it very recently.

2. Suppose galaxy Gaw and galaxy Naegle are separated by a distance of 2400 Mpc (Mpc = megaparsec =  $3.0856 \times 10^{22}$  m) today. Galaxy Gaw emitted some photons long ago which are today being received by funny-looking astronomers in galaxy Naegle.

- (a) The astronomers discover that a hydrogen line normally found at  $\lambda = 650$  nm has been shifted to  $\lambda = 1950$  nm. What is the redshift  $z$  they measure for galaxy Gaw?

From the definition of the redshift  $z$ ,

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{1950 \text{ nm} - 650 \text{ nm}}{650 \text{ nm}} = 2.$$

So Naegle's astronomers see a redshift of  $z = 2$  for galaxy Gaw.

(b) **How far apart were the galaxies when the photon was emitted?**

We know that the wavelength of light is “stretched” along with the expansion of the universe. So since the light waves have been stretched by a factor of  $1950 \text{ nm} / 650 \text{ nm} = 3$ , the universe has also expanded by a factor of 3. Since the distance today is 2400 Mpc, the distance back then must have been  $(2400 \text{ Mpc})/3 = 800 \text{ Mpc}$ . To express this mathematically, we write

$$1 + z = \frac{R_0}{R},$$

where  $R_0$  is some distance today, and  $R$  is the distance at the time corresponding to a given redshift  $z$ . So

$$R = \frac{R_0}{1 + z} = \frac{2400 \text{ Mpc}}{1 + 2} = 800 \text{ Mpc}.$$

(c) **If the distance between the galaxies is increasing at 120,000 km/s today, what is the Hubble constant,  $H_0$ ?**

Hubble's law is that  $v = Hd$ , where  $v$  is the rate at which the distances are increasing. The subscript zero means the value “today” (since the Hubble “constant” can change with time!). So

$$H_0 = \frac{v}{d} = \frac{120,000 \text{ km s}^{-1}}{2400 \text{ Mpc}} = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

(d) **Based on the previous part, what is the Hubble time? Express in years (1 year  $\approx 3.16 \times 10^7$  sec).**

Note that the Hubble constant has units of km upstairs and Mpc downstairs; you can convert these into the same unit and cancel them, leaving 1/time. The Hubble time is defined to be  $t_H = 1/H$ . If the universe were almost empty (with no matter or  $\Lambda$ ), this would be the same as the age of the universe. Universes with matter but no  $\Lambda$  are younger than  $t_H$ , while universes with a  $\Lambda$  can be older.

The Hubble constant from part (b) is

$$H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \times \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \times \left( \frac{1 \text{ Mpc}}{3.0856 \times 10^{22} \text{ m}} \right) = 1.62 \times 10^{-18} \text{ s}^{-1}.$$

So

$$t_H = \frac{1}{H_0} = 6.17 \times 10^{17} \text{ s} \times \left( \frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}} \right) = 1.95 \times 10^{10} \text{ yr} \approx 20 \text{ billion years}.$$

**3. Why do cosmologists now believe the steady-state idea is wrong? Which is more philosophically appealing to you: steady-state, or big bang? Why?**

There are a number of observational tests that the steady-state theory fails. The most damning of these is the observation of the cosmic microwave background (CMB), discovered in 1965. In the big bang paradigm, this radiation is easily explained as the remnant of a time when the universe was hot and dense, so that matter emitted blackbody radiation. The steady-state theory has only extremely contrived explanations for the origin of the CMB, and these ideas are incapable of also explaining the abundances of the elements, which are elegantly understood using the calculations of big bang nucleosynthesis (BBN) plus later formation of the heavier elements in stars. Finally, astronomical observations have shown that the universe *was* indeed different in the past, so that the Perfect Cosmological Principle is manifestly false. The first evidence for this was presented by Martin Ryle in 1955 based on his observations of the distribution of radio sources in the universe. In the 1960s, very luminous sources called *quasars* were discovered, and it has been shown that there abundance

was much greater in the past, at redshifts of 2 or 3. Today, large optical telescopes can directly observe galaxies so distant (beyond redshift 5) that we are seeing them when the universe was only a small fraction of its current age, and they look quite different from the galaxies we see today. It has been suggested that the steady-state theory was popular among some British cosmologists perhaps because they lived in a declining empire, and so it was natural to deny the evolutionary aspects of the universe—in a static universe, nothing changes and you can always bask in your (former) glory!

The class voted by an overwhelming margin (89% to 11%) for the big bang. Most people preferred a universe that is evolving and changing to one that is always basically the same. But be warned, if it expands forever and there is a  $\Lambda$ , it could be a very bleak place in several tens of billions of years!