

ASTRONOMY 9: HISTORY OF COSMOLOGY

Review Worksheet

2000 February 11

Scientific Notation

Large numbers such as 32,000,000 (32 million, approximate population of California) are written as 3.2×10^7 to make life easier. This means you have to move *seven* spaces to the *right* of the decimal point in 3.2 to get 32,000,000. Small numbers like 0.00068 are written 6.8×10^{-4} because you need to move *four* spaces to the *left* of the decimal point in 1.5 to get 0.00015. Generally you want to express quantities as a number between 1 and 10, multiplied by 10 to an integer power. When your calculator or computer says 4.2e-21, it means 4.2×10^{-21} .

Remember that divisions and multiplications are done before additions and subtractions. Make sure you understand the following examples.

- To add or subtract numbers, first make sure the exponents are the same:

$$4 \times 10^4 + 3.2 \times 10^2 = 4 \times 10^4 + 0.032 \times 10^4 = (4 + 0.032) \times 10^4 = 4.032 \times 10^4.$$

- To multiply numbers, add the exponents:

$$(5.1 \times 10^8) \times (4 \times 10^{-3}) = (5.1 \times 4) \times (10^8 \times 10^{-3}) = 20.4 \times 10^{8-3} = 20.4 \times 10^5 = 2.04 \times 10^6.$$

- To divide numbers, subtract the exponents:

$$\frac{1.4 \times 10^{-8}}{2 \times 10^7} = \frac{1.4}{2} \times 10^{-8-7} = 0.7 \times 10^{-15} = 7 \times 10^{-16}.$$

- To exponentiate numbers, multiply the exponents:

$$(2 \times 10^6)^5 = 2^5 \times (10^6)^5 = 32 \times 10^{6 \times 5} = 32 \times 10^{30} = 3.2 \times 10^{31}.$$

Unit Conversion

Here are some common prefixes for units:

- G = giga = billion = 10^9
- M = mega = million = 10^6
- k = kilo = thousand = 10^3
- c = centi = $1/100 = 10^{-2}$
- m = milli = $1/1000 = 10^{-3}$
- μ = micro = $1/1,000,000 = 10^{-6}$

A nice trick for converting units is to remember that you can always multiply a number by 1, without changing its value! Then you construct ratios equal to one and multiply them so that the units you want to get rid of cancel out, and the units you want to get are left over at the end.

For example, suppose we want to convert 60 mph (miles/hour) into meters per second (m/s). We know that 1 hour = 60 min, 1 min = 60 sec, 1 mile \approx 1.6 km, and 1 km = 1000 m. So ratios like this are equal to unity:

$$\frac{1 \text{ hour}}{60 \text{ min}} = \frac{60 \text{ min}}{1 \text{ hour}} = 1.$$

Putting some of these together, we can see how to get the units we want:

$$60 \text{ mph} = \left(60 \frac{\text{miles}}{\text{hour}}\right) \times \left(\frac{1 \text{ hour}}{60 \text{ min}}\right) \times \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) \times \left(\frac{1.6 \text{ km}}{1 \text{ mile}}\right) \times \left(\frac{1000 \text{ m}}{1 \text{ km}}\right).$$

Crossing out the units that cancel and multiplying all the numbers together, we end up with

$$60 \text{ mph} \approx 26.7 \text{ m/s}.$$

So when you're driving down the freeway at 60 mph, you're going about 26.7 meters every second!